

Discrete Structures

Solutions to HW2

1. (a) **F** (b) **T** (c) **T** (d) **T** (e) **F** (f) **T** (g) **T** (h) **F** (i) **F** (j) **F**

Explanations and comments (not required):

- (a) Every $x \in (0, 1]$ is a counterexample. In this case, every $y > x$ satisfies $y > x^2$.
 (b) The value $x = 2$ is an example (a witness). In fact, every $x \geq 1$ is a witness.
 (c,d) Whenever $x < y$, the value $z = \frac{x+y}{2}$ satisfies $x < z < y$.
 (e) Whenever $x < y$, there is no number less than x and also larger than y .
 (f) Whenever $x < y$, the value $z = x - 1$ satisfies the required conditions.
 (g) If $x < y < 1$, then $x < 1$.
 (h) We are free to choose $z = x + 1$, so that every $y < x$ satisfies $y < x < z$.
 (i) See (h) (sorry for the repeated problem).
 (j) Assuming $y < x$, we are free to choose $z = \frac{x+y}{2}$, so that $y < z < x$.
2. (a) **$(0, \infty)$**
 (b) **$(0, \infty)$** . Note that the answers to 2(a), 2(b) and 2(c) are identical. No credit for including either of the endpoints 0 or ∞ .
 (c) $\overline{\bigcap_{n=1}^{\infty}(-\infty, \frac{1}{n})} = \overline{(-\infty, 0]} = \mathbf{(0, \infty)}$.
 (d) **$\mathbb{R}$** . It is also correct to write this in interval notation as **$(-\infty, \infty)$** .
 (e) $\bigcup_{m=0}^{\infty} \bigcap_{n=m+1}^{\infty} [m, n] = \bigcup_{m=0}^{\infty} [m, m+1) = \mathbf{[0, \infty)}$.
 (f) $\bigcap_{m=0}^{\infty} \bigcup_{n=m+1}^{\infty} [m, n] = \bigcap_{m=0}^{\infty} [m, \infty) = \emptyset$. This could also be written as **$\{ \}$** .
3. **$(I) \equiv (b)$; $(II) \equiv (f)$; $(III) \equiv (a)$; $(IV) \equiv (c)$; $(V) \equiv (d)$; $(VI) \equiv (e)$** .

For example, the first equivalence follows from applying De Morgan's laws and other propositional equivalences we have seen:

$$\begin{aligned}
 \neg(\forall x)(\forall y)(P(x) \rightarrow Q(y)) &\equiv (\exists x)\neg(\forall y)(P(x) \rightarrow Q(y)) \\
 &\equiv (\exists x)(\exists y)\neg(P(x) \rightarrow Q(y)) \\
 &\equiv (\exists x)(\exists y)\neg((\neg P(x)) \vee Q(y)) \\
 &\equiv (\exists x)(\exists y)(P(x) \wedge (\neg Q(y))).
 \end{aligned}$$

4. **(a) , (c) , (d) , (g) , (h)** are countable (and (b) , (e) , (f) are uncountable.)