

Discrete Structures

HW3

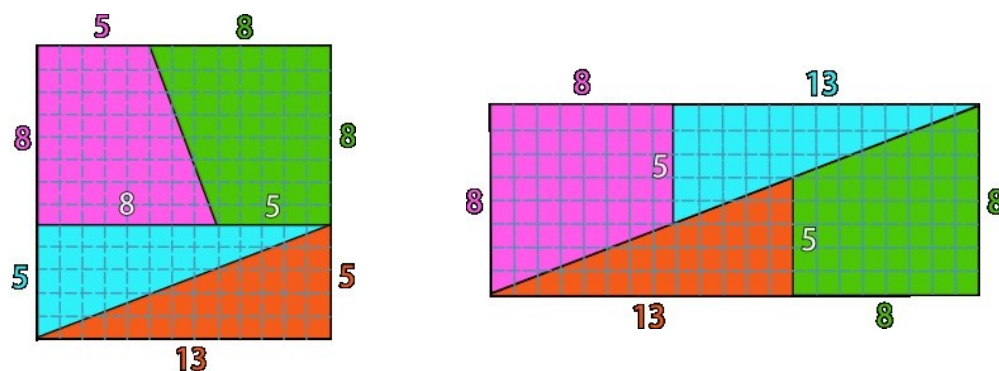
(Due 5:00pm, Thursday, October 15, 2026, on WyoCourses)

See the syllabus for expectations regarding homework submissions. In particular, you are reminded that while discussion with other students is encouraged, your submissions must be your own work, not copied from any other sources, human or otherwise. You are also reminded that organization, presentation and clarity of solutions are all considered; proper notation, spelling, grammar, vocabulary and punctuation are required. Each question is worth 10 points, for a total of 60 points.

Instructions. In each of #1–4, you are given a ‘Theorem’ and an incorrect ‘Proof’. In each case, determine the flaw in the reasoning. (You are *not* being asked to show that the statement of the ‘Theorem’ is incorrect, only that the ‘Proof’ is incorrect.)

1. **‘Theorem’.** $169 = 168$.

‘Proof’. A square (13 units by 13 units in size) is shown on the left. We decompose the square into four pieces as shown, which we then reassemble to form the rectangle (8 units tall, 21 units wide) shown on the right. Comparing areas gives $169 = 168$.



2. **‘Theorem’.** $5 = -3$.

‘Proof’. From $5 = -3$, we subtract 1 from both sides to get $4 = -4$; then square both sides to get $16 = 16$. This is correct.

3. **‘Theorem’.** There are no integer solutions to the equation $42 = x^3 + y^3 + z^3$.

‘Proof’. If (x, y, z) is an integer solution, then each of x, y, z is at most 3; otherwise the right side has a term greater than or equal to $4^3 = 64$, exceeding the left hand side, a contradiction. So $x, y, z \in \{0, 1, 2, 3\}$, and checking all possible cases for x, y, z in this range gives $x^3 + y^3 + z^3 \in \{0, 1, 2, 3, 8, 9, 10, 16, 17, 24, 27, 28, 29, 35, 36, 43, 54, 55, 62, 81\}$. This rules out 42 as a possible value for the right hand side.

4. **‘Theorem’.** There are at most two pairs of integers (x, y) satisfying the equation $x^2 = 61y^2 + 1$.

‘Proof’. We have two solutions $(x, y) = (\pm 1, 0)$ and there cannot be any others, since the equation to be solved is quadratic, and therefore has at most two real roots.

Instructions. In each of #5,6, you are asked to write your own proof, either for existence or for nonexistence of integer solutions.

5. Do there exist integers m and n such that the last two decimal digits of $m^2 + n^2$ are 43? Prove your assertion.
6. Do there exist *positive* integers m and n such that $m^2 = 10n^2 + 1$?