

Name

UNIVERSITY
OF WYOMING

COSC 2300—Fall 2026

Computer Science

Discrete Structures

Quiz 3

Thursday, October 6, 2026

Fill in the blanks below using words selected from the following list:

if	or	proof	contradiction	empty	rational	multiple
then	even	because	contrapositive	equal	rational	positive
and	odd	therefore	tautology	converse	integers	positives

to make the following a valid proof. (Some words from this list may be used more than once, and some not at all.)

Theorem. The equation $m^2 - n^2 = 4k + 2$ has no solution in integers.

Proof. Suppose m, n, k are **integers** satisfying $m^2 - n^2 = 4k + 2$. Observe that $m + n$ and $m - n$ have the same parity, i.e. both are even **or** both are **odd**. This is because $m + n = (m - n) + 2n$ where $2n$ is **even**.

Case (i). $m + n$ and $m - n$ are both odd. Then $m^2 - n^2 = (m + n)(m - n)$ is a product of two **odd** numbers, which is odd. But the right hand side $4k + 2$ is even, a contradiction.

Case (ii). $m + n$ **and** $m - n$ are both **even**. Then $m^2 - n^2 = (m + n)(m - n)$ is a product of two even numbers, which is a multiple of 4. But the right hand side $4k + 2$ is clearly not a **multiple** of 4, again a **contradiction**.

In both cases, we obtain a **contradiction**. This proves the statement of the theorem. ☐