Name Solution Key

Section ☐ **01** Earl-8am ☐ **02** Yeung-9am ☐ **03** Furtado-10am ☐ **04** Li-11am
☐ **05** Furtado-11am ☐ **06** Zhong-12noon ☐ **07** Wiseman-1:10pm ☐ **08** Yeung-2:10pm

UNIVERSITY OF WYOMING — Math 2200 — Spring 2019 — Department of Mathematics

BRIGGS COCHRAN GILLETT **Calculus I**

$\frac{d}{dx}\sin x = \cos x$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Common Exam II

5:15–7:00pm Thursday March 28, 2019

Instructions. Indicate your name and section/instructor above. You may use a scientific non-graphing calculator—no other aids are allowed. Cell phones and other devices must be turned off and left in your backpack/bag during the exam. **Write clearly**, using good mathematical notation and showing all required steps in the space provided. *Unless otherwise stated, justify your answers.* A list of useful formulas appears on the last page. Total value: 100 points.

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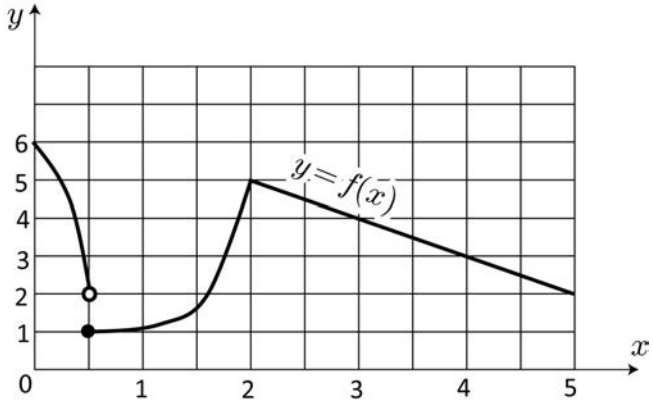
1. *(12 points)* Use the graph of $f(x)$ shown on the right to answer A,B,C,D. In parts A,B,C, write your answers in the blanks provided.

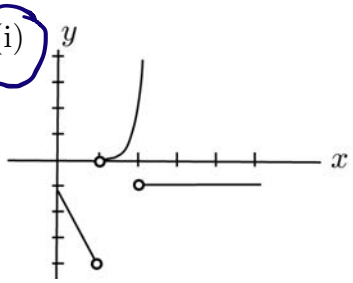


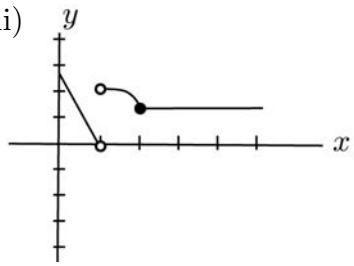
0.5 A. Give the value(s) of x in the interval $(0,5)$ at which at which $f(x)$ is not continuous.

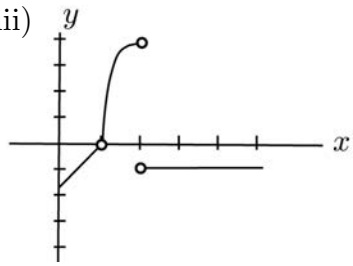
0.5, 2 B. Give the value(s) of x in the interval $(0,5)$ at which at which $f(x)$ is not differentiable.

−1 C. What is the value of $f'(3)$?

D. Which of the following could be a graph of f'? Circle the correct answer.

(i) 

(ii) 

(iii) 

2. *(10 points)* The table below gives selected values of differentiable functions f, f', g, g'.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-7	-1	2	17
1	15	14	11	2
2	3	4	4	7
3	13	5	8	3
4	-9	-3	-5	-4
5	-5	1	4	8

(a) If $h(x) = 2f(x) + 3g(x)$, determine $h'(2)$.

$h'(2) = 2f'(2) + 3g'(2) = 2 \cdot 4 + 3 \cdot 7 = 29$

(b) If $u(x) = f(x)g(x)$, determine $u'(2)$.

$u'(2) = f'(2)g(2) + f(2)g'(2) = 4 \cdot 4 + 3 \cdot 7 = 37$

(c) If $v(x) = \dfrac{f(x)}{g(x)}$, determine $v'(2)$.

$v'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{g(2)^2} = \dfrac{4 \cdot 4 - 3 \cdot 7}{4^2} = -\dfrac{5}{16}$

(d) If $w(x) = g(f(x))$, determine $w'(2)$.

$w'(2) = g'(f(2))f'(2) = g'(3) \cdot 4 = 3 \cdot 4 = 12$

(e) If $r(x) = \big(f(x) - g(x)\big)^3$, determine $r'(2)$.

$r'(2) = 3(f(2) - g(2))^2 (f'(2) - g'(2)) = 3(3-4)^2(4-7) = -9.$

3. *(30 points)* In each case, find the required derivative. *Simplification is not required.*

(a) If $f(x) = x^4 + 6x^{5/2} - 3x^{-2}$ then

$f'(x) =$ $4x^3 + 15x^{3/2} + 6x^{-3}$

(b) If $f(x) = \cos^5(x)$ then

$f'(x) =$ $(5\cos^4 x)(-\sin x) = -5\sin x \cos^4 x$

(c) If $g(t) = e^{t\sin(t)}$ then

$g'(t) =$ $e^{t\sin t}(\sin t + t\cos t)$

(d) If $H(x) = (3x+1)\sqrt{x}$ then

$H'(x) =$ $3\sqrt{x} + (3x+1)\cdot\frac{1}{2\sqrt{x}} = \frac{9x+1}{2\sqrt{x}}$

(e) If $G(x) = \dfrac{2x-1}{(x+4)^3}$ then

$G'(x) =$ $\dfrac{(x+4)^3\cdot 2 - 3(x+4)^2(2x-1)}{(x+4)^6} = \dfrac{11-4x}{(x+4)^4}$

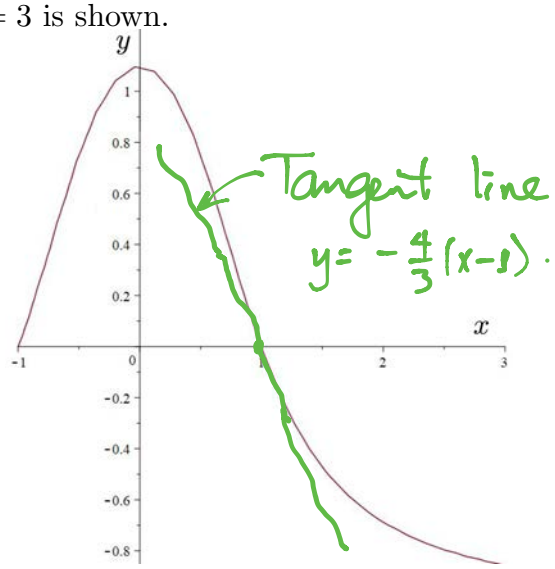
(f) $\dfrac{d}{dx}\sin^{-1}(3-x) = \dfrac{1}{\sqrt{1-(3-x)^2}}\cdot(-1) = \dfrac{-1}{\sqrt{(4-x)(x-2)}}$

4. *(10 points)* Let $f(x) = \ln(2x - 4)$. Find and *simplify*:

(a) $f'(x) = \frac{1}{2x-4} \cdot 2 = \frac{1}{x-2}$

(b) $f''(x) = -\frac{1}{(x-2)^2}$

5. *(12 points)* A graph of the relation $e^y + 2(y+1)x^2 = 3$ is shown.



(a) Does the point $(1, 0)$ satisfy the relation?

Yes: $2^0 + 2 \cdot 1 \cdot 1 = 3$.

(b) Find the slope of the tangent line to the graph at the point $(1, 0)$.

Take $\frac{d}{dx}$ on both sides of $e^y + 2(y+1)x^2 = 3$ to obtain

$e^y \frac{dy}{dx} + 2\frac{dy}{dx} \cdot x^2 + 2(y+1) \cdot 2x = 0$. Evaluate at $(1,0)$ to get

$m + 2m + 4 = 0$; $m = -\frac{4}{3}$ where $m = \left.\frac{dy}{dx}\right|_{(1,0)}$.

(c) Determine the equation of the tangent line to the graph at the point $(1, 0)$.

$y - 0 = m(x-1)$, i.e. $y = -\frac{4}{3}(x-1)$.

(d) In the graph given above, sketch and label the tangent line found in (c).

6. *(10 points)* A ball is thrown upwards and has height above the ground (in feet) given by

$$s(t) = -16t^2 + 160t$$

at time t (in seconds).

(a) Determine the velocity $v(t)$ of the ball at time t. What are the correct units of $v(t)$?

$v(t) = s'(t) = -32t + 160$,
the velocity in ft/sec.

(b) Determine the acceleration $a(t)$ of the ball at time t. What are the correct units of $a(t)$?

$a(t) = v'(t) = -32$ ft/sec^2

(c) At what time(s) does the velocity of the ball equal zero?

$v(t) = 0$ at $t = 5$ sec.

(d) At what time(s) is the ball at ground level?

$s(t) = 0$ at $t = 10$ sec.

7. *(8 points)* The line $y = 3x - 5$ is tangent to the graph of a function $y = f(x)$ at the point $(2, 1)$. Determine the following values:

$f(2) =$ 1

$f'(2) =$ 3 as given by the slope of the tangent line.

8. *(8 points)* Determine the indicated derivatives *in simplified form*:

(a) $\frac{d}{dx}e^e =$ 0 (noting that e^e is a constant)

(b) $\frac{d}{dx}e^x =$ e^x

(c) $\frac{d}{dx}x^e =$ ex^{e-1}

(d) $\frac{d}{dx}e^{ex} =$ $ee^{ex} = e^{ex+1}$