University of Wyoming

Math 2850—Fall 2025

Putnam Team Seminar

Department of Mathematics

Practice Problems 12

Monday, November 24, 2025

1. Let D_n be the determinant of order n of which the element in the ith row and the jth column is the absolute value of the difference of i and j. Show that D_n is equal to
$$(-1)^{n-1}(n-1)2^{n-2}.$$

2. Let $f(x, y)$ be a continuous, real-valued function on $\mathbb{R}^2$. Suppose that, for every rectangular region R of area 1, the double integral of $f(x, y)$ over R equals 0. Must f be identically 0?

3. Is there a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$ such that $f'(x) = f(f(x))$ for all x?

4. Let $P(x)$ be a polynomial of degree n such that $P(x) = Q(x)P''(x)$, where $Q(x)$ is a quadratic polynomial and $P''(x)$ is the second derivative of $P(x)$. Show that if $P(x)$ has at least two distinct roots then it must have n distinct roots.

5. Determine all positive integers N for which the sphere
$$x^2 + y^2 + z^2 = N$$
has an inscribed regular tetrahedron whose vertices have integer coordinates.

6. A class with $2N$ students took a quiz, on which the possible scores were $0, 1, \ldots, 10$. Each of these scores occurred at least once, and the average score was exactly 7.4. Show that the class can be divided into two groups of N students in such a way that the average score for each group was exactly 7.4.

7. Four points are chosen uniformly and independently at random in the interior of a given circle. Find the probability that they are the vertices of a convex quadrilateral.