Relatively Prime Numbers

Given two large integers, what is the probability that they are relatively prime? More precisely, if integers m, n are randomly chosen in the interval $[0, L]$ where L is large, what is the probability that $\gcd(m, n) = 1$? We conduct T trials and count r, the number of times that m and n are relatively prime, finally computing the ratio $\frac{r}{T}$. In the limit as $T, L \to \infty$, this ratio $\frac{r}{T} \to \frac{1}{\zeta(2)} = \frac{6}{\pi^2} \approx 0.607927$ where $\zeta(x)$ is the Riemann zeta function. Let's test this fact:

First specify the size of our interval and the number of trials:

```
L=10^100; Print["Choose pairs of integers in the interval [0,",L,"]."]
T=1000000; Print["Conduct ",T," trials."]
```

```
Choose pairs of integers in the interval [0,
 10 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 -
  000 000 000 000 000].
Conduct 1 000 000 trials.
```

```
r=0;
For[t=0, t<T, t++,
   m=RandomInteger[L];
   n=RandomInteger[L];
   r=r+If[GCD[m,n]==1,1,0]];
Print["m and n were relatively prime ",1.0*(r/T)," of the time."]
```

```
m and n were relatively prime 0.608694 of the time.
```

```
p = 0.608694
```

Out[•]=

```
0.608694
```

This gives us an experimental estimate for π !

```
Sqrt[6 / p]
```

Out[•]=

```
3.13961
```

To improve the accuracy of this estimate, increase the size L of the interval, and the number of trials T.