HW1

(Due 5:00pm Friday, March 7, 2025, on WyoCourses)

> *Instructions:* Work by hand and calculator showing your work. Appropriate computer software (such as Maple, Sage or Mathematica) may then be used to *check* answers. Submit solutions through WyoCourses. See the syllabus and FAQ for general expectations regarding homework.

In the following, consider two Pythagorean triples (a, b, c) to be *essentially the same* if one is the same as the other up to reordering. For example, $(6, 8, 10)$ and $(8, 6, 10)$ are essentially the same; but $(6, 8, 10)$ and $(3, 4, 5)$ are essentially different.

1. (*20 points*) How many essentially different Pythagorean triples are there containing the number 40? List them all. How many of these triples are primitive? Justify all steps that are not obvious.

2. (*20 points*) Show that every positive integer (with exactly two exceptions) is contained in some Pythagorean triple.

3. (*20 points*) In the following, work by hand, giving the *simplest* answer available.
   (a) Using Euclid's Algorithm (performed by hand), find integers r, s such that $631r + 101s = 1$.
   (b) Using (a), find the inverse of 631 mod 101. Answer in simplified form, expressed in the range $\{0, 1, 2, \ldots, 100\}$.

4. (*20 points*) Solve the linear system $28x + 71y = 38$, $91x + 47y = 22$ in $\mathbb{F}_{101}$. Here $\mathbb{F}_{101} = \{0, 1, 2, \ldots, 100\}$ is the field of integers mod 101. Your answers should be in simplified form; e.g. $\frac{51}{13} = 35$ where 35 is simplified but $\frac{51}{13}$ is not). Whenever division in $\mathbb{F}_{101}$ is required, use Euclid's Algorithm by hand as in #3.

5. (*20 points*) Write each of the following as a sum of two integer squares:
   (a) 89
   (b) 137
   (c) $12193 = 89 \cdot 137$. In this case there are essentially two different solutions; give both of them.

6. (*20 points*) Write each of the following in the form $x^2 + 7y^2$ where x, y are integers:

(a) 43

(b) 79

(c) $3397 = 43 \cdot 79$. In this case there are essentially two different solutions; give both of them.