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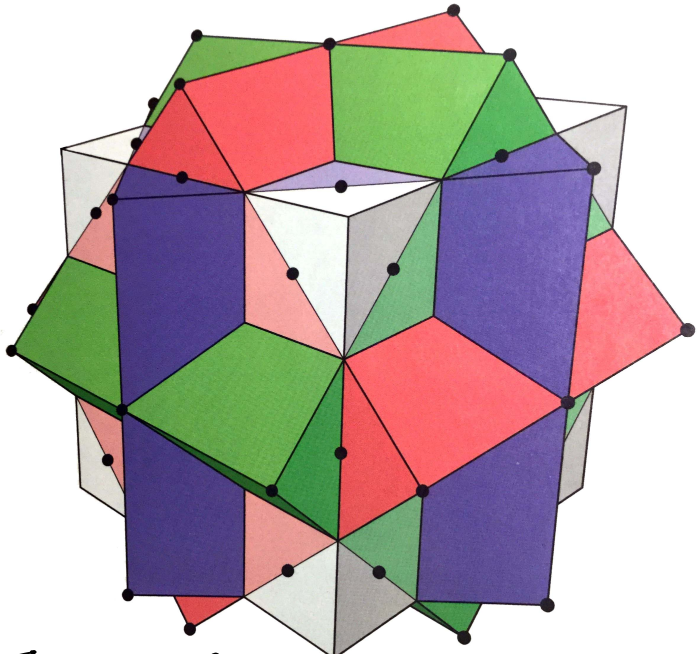

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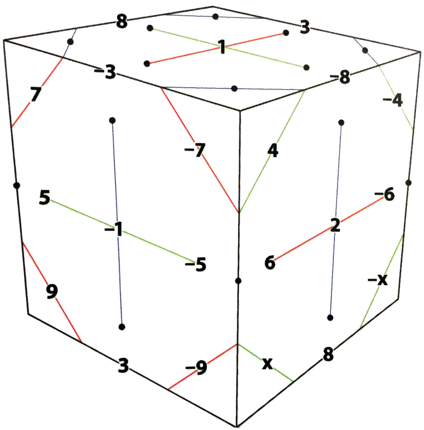

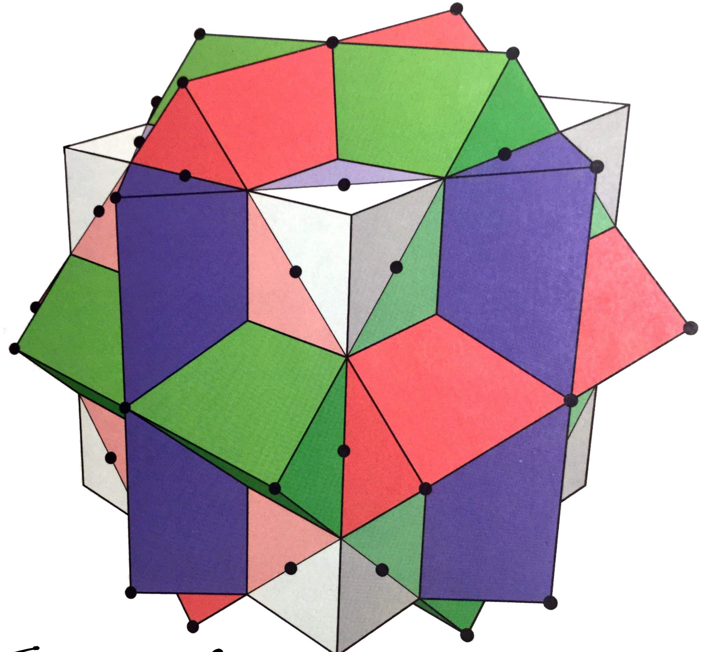

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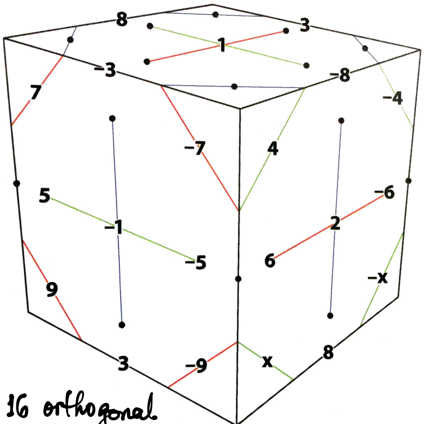

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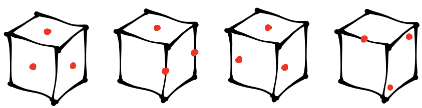

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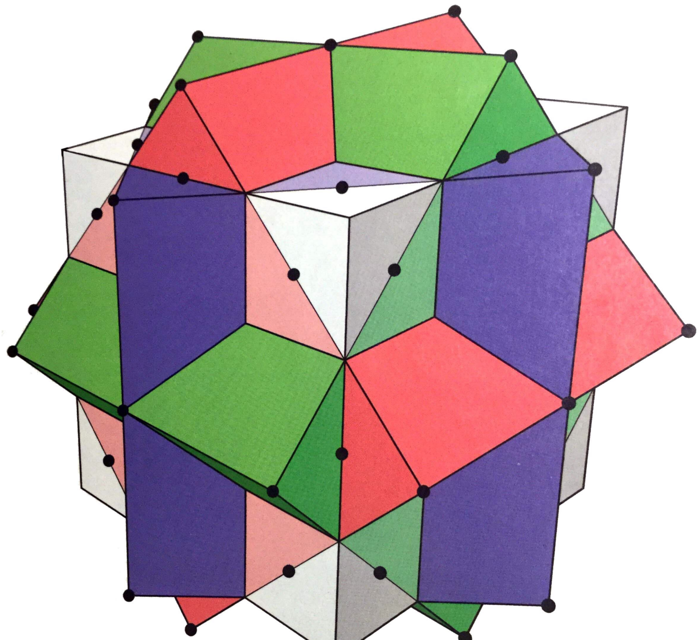

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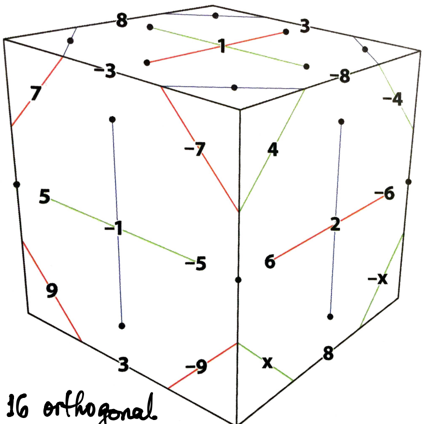

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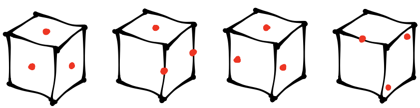

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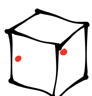
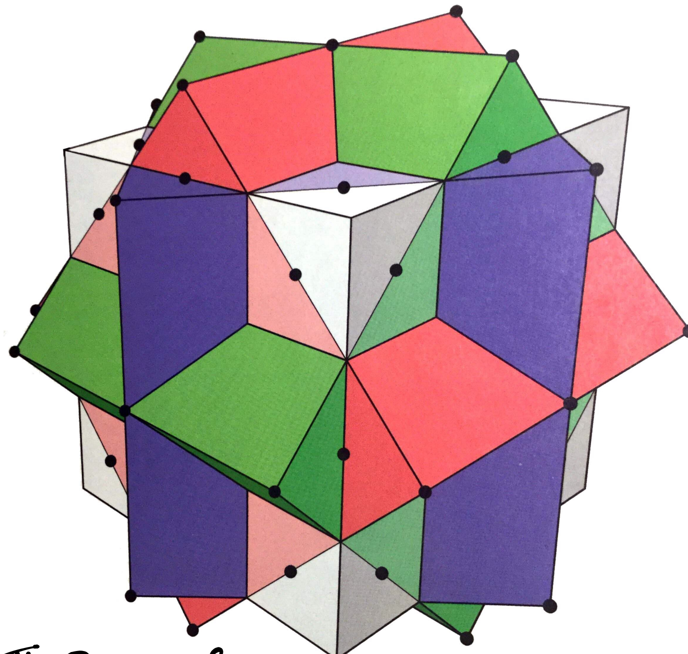
Another 24 pairs of orthogonal lines: 



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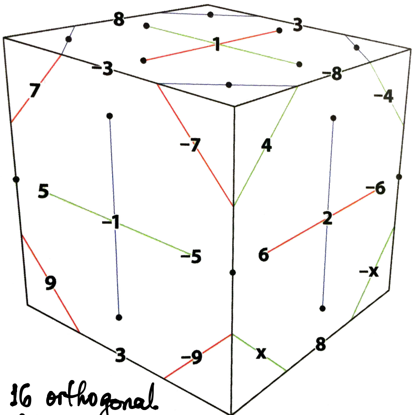

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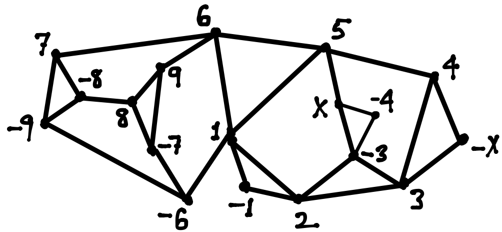

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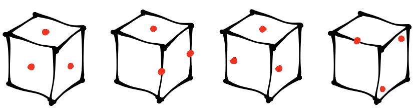

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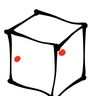

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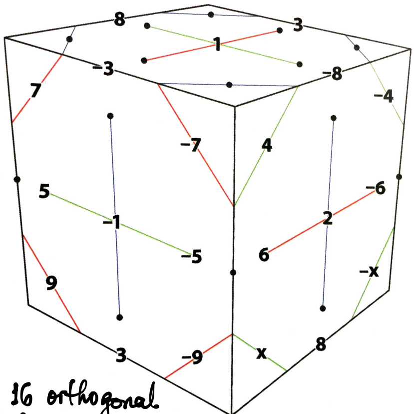

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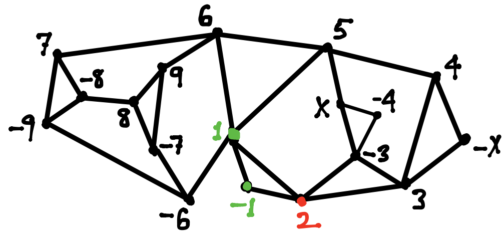

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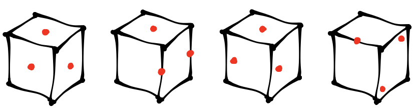

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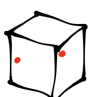

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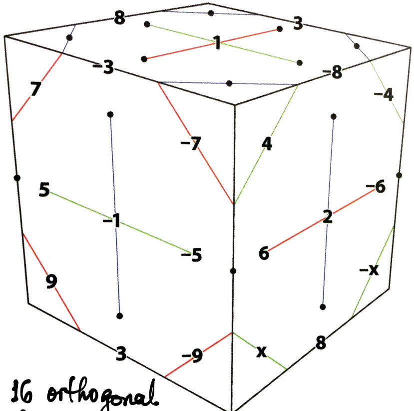

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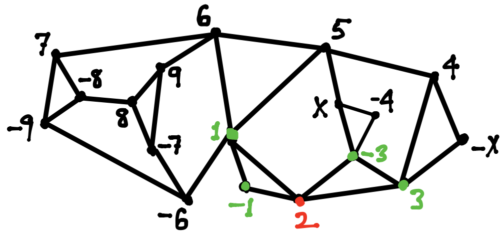

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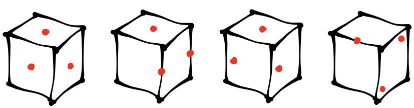

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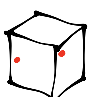

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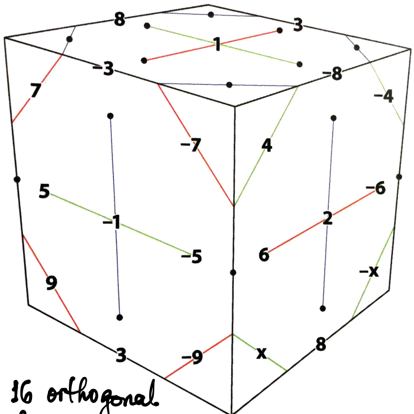

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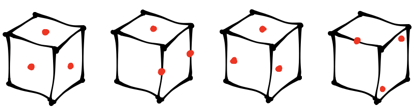

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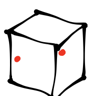

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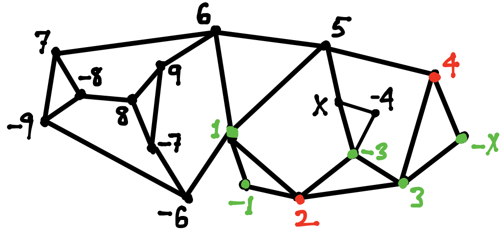

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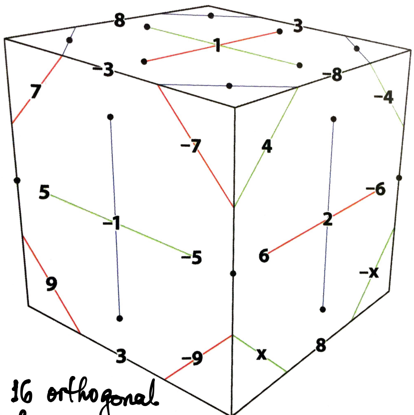

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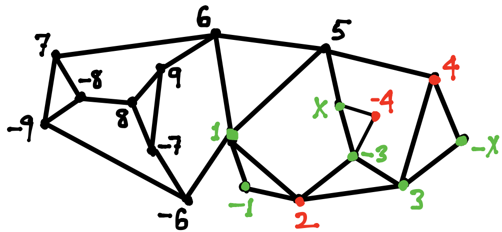

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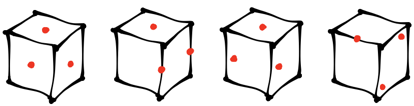

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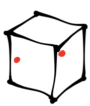

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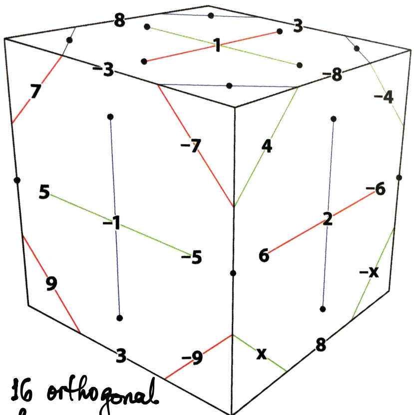

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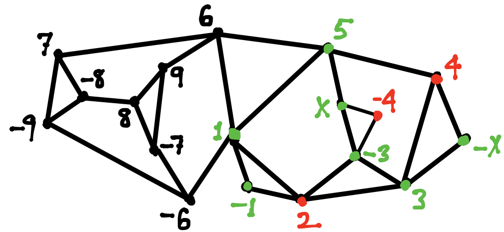

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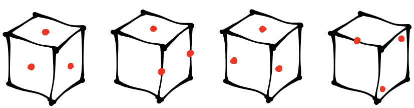

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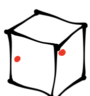

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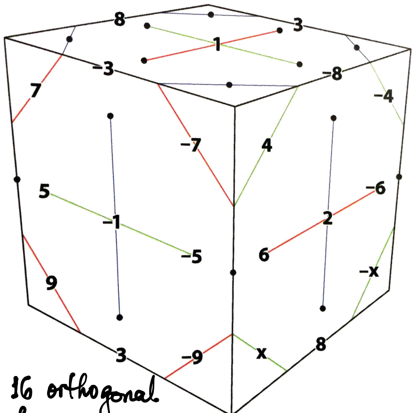

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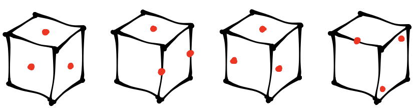

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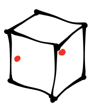

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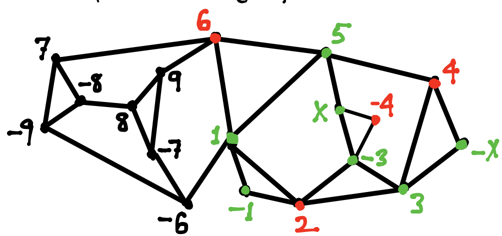

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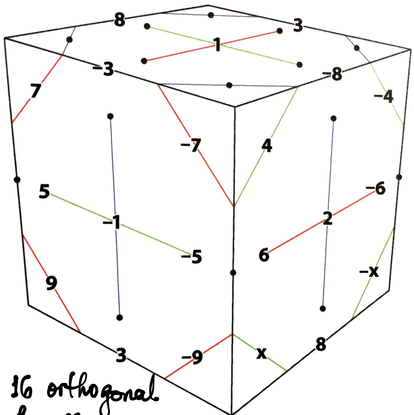

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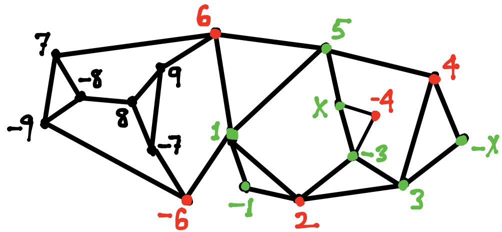

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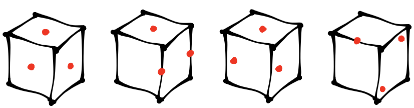

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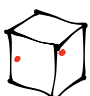

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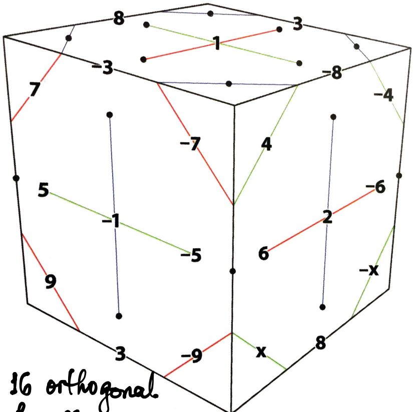

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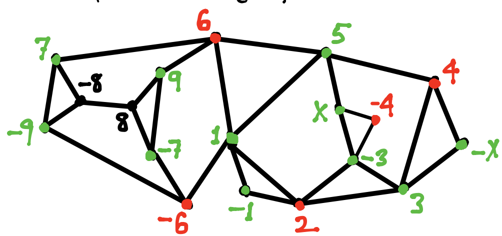

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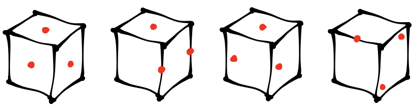

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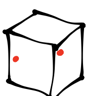

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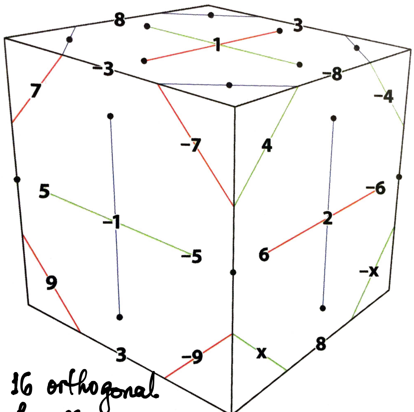

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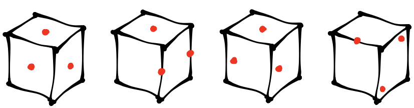

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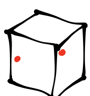

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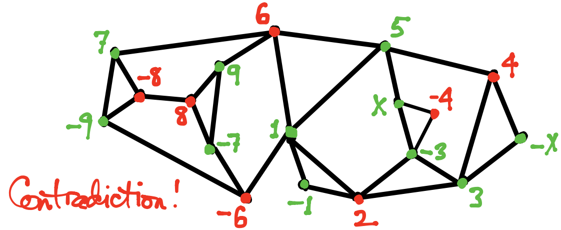

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SPIN Axiom. *Measurements of the squared (components of) spin of a spin 1 particle in three orthogonal directions always give the answers* 1, 0, 1 *in some order.*

The TWIN Axiom. *For twinned spin 1 particles, suppose experimenter A performs a triple experiment of measuring the squared spin component of particle a in three orthogonal directions x, y, z, while experimenter B measures the twinned particle b in one direction, w. Then, if w happens to be in the same direction as one of x, y, z, experimenter B's measurement will necessarily yield the same answer as the corresponding measurement by A.*

The MIN Axiom. *Assume that the experiments performed by A and B are spacelike separated. Then, experimenter B can freely choose any one of the 33 particular directions w, and a's response is independent of this choice. Similarly and independently, A can freely choose any one of the 40 triples x, y, z, and b's response is independent of that choice.*

The Free Will Theorem. *The axioms SPIN, TWIN, and MIN imply that the response of a spin 1 particle to a triple experiment is free—that is to say, is not a function of properties of that part of the universe that is earlier than this response with respect to any given inertial frame.*